ASTRONOMY QUALIFYING EXAM
January 2013

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

a. (7 points) Assume that the gas component of a galaxy, with a mass fraction f_g, is virialized and follow the overall density profile of the galaxy, a singular isothermal sphere mass profile,

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}, \tag{1}$$

where σ is the velocity dispersion of the galaxy. The galaxy has a central AGN, which is radiating at the Eddington luminosity,

$$L_{Edd} = 4\pi G c m_p M_{BH}/\sigma_T. \tag{2}$$

A fraction, f_w, of the energy radiated by the central AGN is deposited to the gas in the form of kinetic energy. This kinetic "feedback" energy from the AGN can drive the gas in the host galaxy to flow outward. Assume that the final gas outflow is in a spherical shell with a constant velocity, v, and half of the kinetic feedback energy is converted to the gravitational potential of the gas and the other half to the kinetic energy of the gas during the outflowing process. Use the conservation or transfer of energy to show that the final gas wind speed is

$$v^3 = \frac{G L_{Edd} f_w}{2\sigma^2}. \tag{3}$$

b. (3 points) If the wind speed is large enough to escape the potential well of the galaxy ($v = \sigma$), the central AGN will blow out the majority of gas in the galaxy and terminate the formation of stars. Show that this gives us the M_{BH}–σ relation,

$$M_{BH} = \frac{1}{2\pi} \frac{\sigma_T}{G^2 c m_p} \frac{1}{f_w} \sigma^5, \tag{4}$$

where G is the gravitational constant, σ_T is the Thomson cross section, c is the speed of light, and m_p is the mass of a proton.

PROBLEM 2

The Universe is dominated by dark energy today, but for a rough estimate of the age of the Universe at $2 < z < 100$, we can assume a matter dominated universe.

The Friedman Equation is

$$\dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2, \tag{5}$$

where

$$\frac{8\pi G}{3}\rho = \Omega H^2. \tag{6}$$

(1) Derive the formula for the age of a matter-dominated universe at redshift z, assuming that we know t_0 (the age of the Universe today). (5 pts)

(2) What is the current measurement of t_0? Estimate the age of the Universe at $z = 10$ using this information. (2 pts)

(3) How does dark energy change the age of the Universe today, compared to a flat universe with matter only? (3 pts)

PROBLEM 3

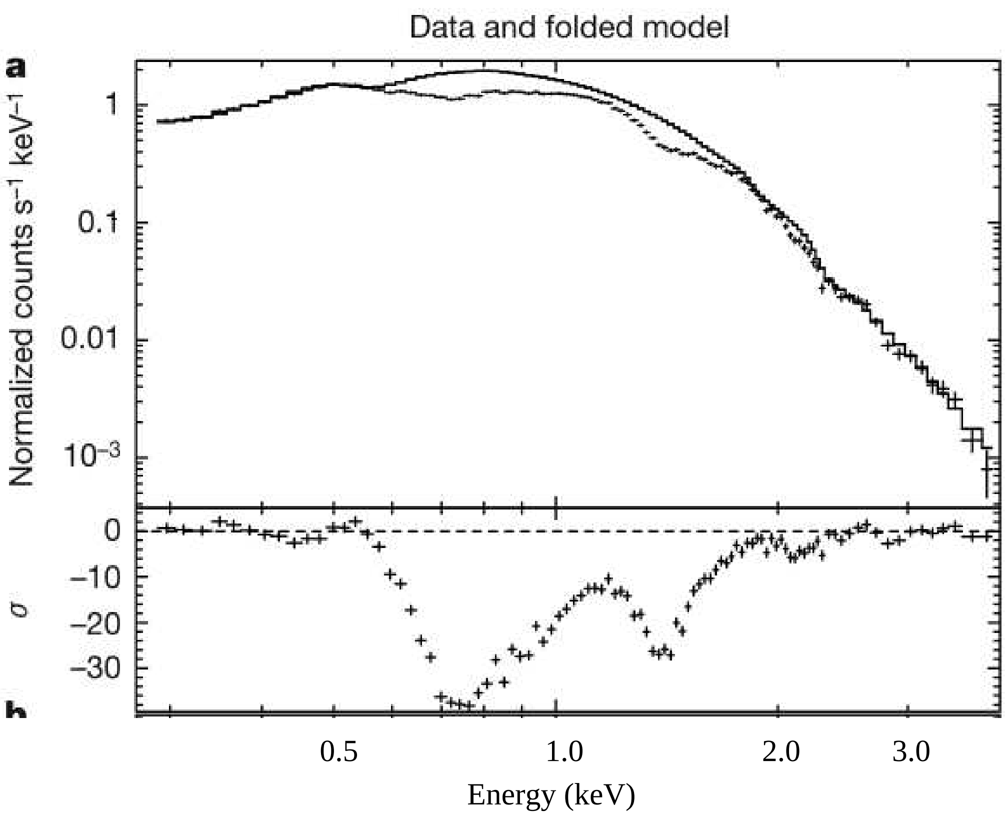
a. An electron in an electromagnetic field will experience a Lorentz force. Write down the equation for the Lorentz force. (2 points).

b. Consider an electron in a uniform magnetic field with a velcocity v. What is the frequency of light emitted by this electron if the velocity vector is oriented perpendicular to the magnetic field lines? (2 points)

c. The figure below shows the X-ray spectrum of an isolated neutron star. Direct your attention to the lower panel, which shows the difference between the spectrum and a blackbody continuum model. Three (possibly 4) absorption lines are seen. Please estimate the frequencies (in Hz) of these absorption lines. Which one is the fundamental frequency and which are harmonics? (2 points)

d. Estimate the magnetic field strength, in gauss, of the neutron star, ignoring general relativistic effects. (2 points)

e. Neutron stars are very compact, and general relativity should not be ignored. GR will affect the frequency of the absorption feature. Will the real feature have a higher frequency or lower frequency than estimated in part (d)? Explain. (2 points)



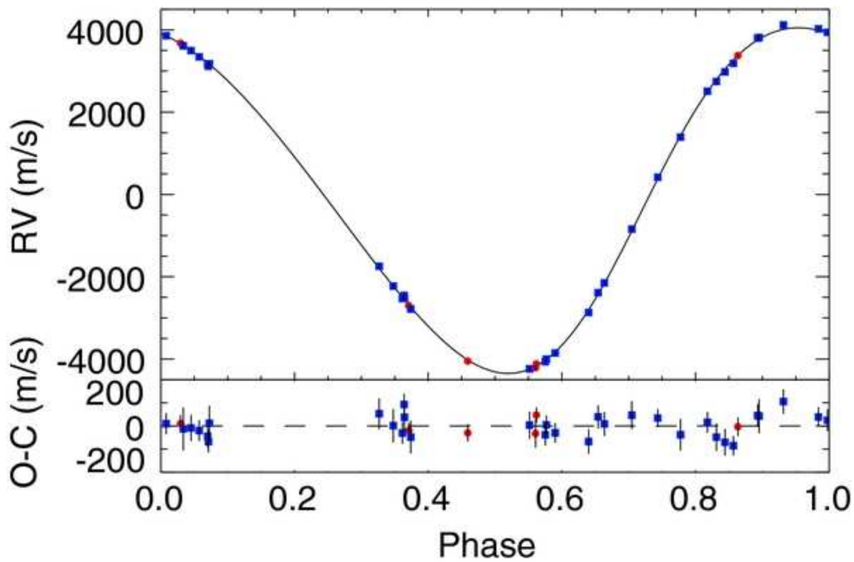
PROBLEM 4

Briefly define and discuss the relevance of the following terms to modern astronomy. 1 point per question

1. Cepheid variable star
2. Initial mass function
3. tunneling in the context of the PPI chain reaction
4. age-metallicity relation
5. damped Lyα system (DLA)
6. s-process
7. G dwarf problem
8. Tully-Fisher relation
9. Galactic thin disk
10. isophotal radius

PROBLEM 5

The following radial velocity phase curve is observed for a companion orbiting a star. Assume e=0 and P=79 days:



a. (7 pts) Derive a general expression for the companion mass.

b. (1 pt) What is the minimum mass of the companion, assuming the host star is a Solar analog?

c. (1 pt) What is the semi-major axis, a, of the companion in AU? Assume sini = 1 and the host star is a Solar analog.

d. (1 pt) The companion is observed to transit the primary star, producing a 2% drop in flux. Assuming the primary is a Solar analog, what is the radius of the companion in R_{Sun}?

PROBLEM 6

1. (4 pts) Show that the formal solution of the plane-parallel radiative transfer equation can be written:

$$\mathsf{I}_\nu(\tau_1,\mu) = \mathsf{I}_\nu(\tau_2,\mu)\mathsf{e}^{-(\tau_2-\tau_1)/\mu} + \frac{1}{\mu}\int_{\tau_1}^{\tau_2} \mathsf{S}_\nu(\mathsf{t})\mathsf{e}^{-(\mathsf{t}-\tau_1)/\mu}\,\mathsf{d}\mu \tag{7}$$

where $S_\nu(t)$ is the source function, $\tau_{1,2}$ are optical depth points in the atmosphere, and μ is the cosine of the angle of the ray.

2. (2 pts) Apply Eqn. 7 to an arbitrary point in the atmosphere of a semi-infinite slab to find:

$$\mathsf{I}_\nu(\tau,\mu) = \int_\tau^\infty \mathsf{S}_\nu(\mathsf{t})\mathsf{e}^{-(\mathsf{t}-\tau)/\mu}\,\mathsf{dt}/\mu \quad \text{for } 0 \le \mu \le 1 \tag{8}$$

$$\mathsf{I}_\nu(\tau,\mu) = \int_0^\tau \mathsf{S}_\nu(\mathsf{t})\mathsf{e}^{-(\tau-\mathsf{t})/(-\mu)}\,\mathsf{dt}/(-\mu) \quad \text{for } 1 \le \mu \le 0 \tag{9}$$

3. (2pts) Integrate Eqns 8 and 9 over angle to find

$$\mathsf{J}_\nu(\tau) = 1/2\left[\int_\tau^\infty \mathsf{dt}\,\mathsf{S}_\nu(\mathsf{t})\int_1^\infty \mathsf{dw}\,\mathsf{e}^{-\mathsf{w}(\mathsf{t}-\tau)}/\mathsf{w} + \int_0^\tau \mathsf{dt}\,\mathsf{S}_\nu(\mathsf{t})\int_1^\infty \mathsf{dw}\,\mathsf{e}^{-\mathsf{w}(\tau-\mathsf{t})}/\mathsf{w}\right] \tag{10}$$

These integrals are of standard form (the first exponential integral):

$$\mathsf{E}_1(\mathsf{x}) = \int_1^\infty \mathsf{e}^{-\mathsf{xt}}/\mathsf{t}\,\mathsf{dt}$$

4. (1 pt) Show that in terms of E_1, J may be written:

$$\mathsf{J}_\nu(\tau) = 1/2\int_\tau^\infty \mathsf{dt}\,\mathsf{S}_\nu(\mathsf{t})\mathsf{E}_1(|\mathsf{t}-\tau|)$$

5. (1 pt) Explain the nature of this final operator.